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Computer Experiments on Ion-Beam Neutralization with Initially Cold Electrons

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Trajectories of electrons and ions shot into an initially field-free space, as in the ion-propulsion problem, have been obtained using a one-dimensional model. The results of a set of computer experiments are presented, in which the ratios of electron-to-ion injection velocities and currents are varied for large but finite ratios of ion-to-electron mass. For electron-to-ion velocity ratios less than about 2, static theories of neutralization are confirmed. For unneutralized and partially neutralized beams, oscillations, and in some cases, randomization of electron trajectories are observed. For electron-to-ion velocity ratios greater than 2 and electron-to-ion current ratios greater than a critical value, neutralization is obtained by means of an oscillating electron sheath that feeds electron current to the ion beam in a self-compensating manner.

I. Introduction

The problem considered here is that of a semi-infinite region bounded by a single perfectly conducting injection plane at zero potential. Electrons and ions in the form of two interpenetrating streams are shot into this region. Each stream has a single velocity on injection. The current in each stream and the velocities of the streams are the variables. If the electron and ion currents are equal in absolute value, we have the usual formulation of the ion-propulsion problem, for which the static theory, subject to certain approximations, has been given.[1–4] If either the electron current or the ion current is zero, we have the single-species diode problem with one electrode at infinity. The boundary condition at infinity in either problem is that the potential is finite or that the field is zero. Fay, Samuel, and Shockley[5] give the solution to the static single-species problem as their type A potential distribution for a finite diode. It applies without modification to the infinite diode considered here. For ratios of currents other than zero or unity, the static theory has not been published, to the best of the authors' knowledge. As will be shown, when there is only partial neutralization and some particles return to the injection plane, the problem is an essentially time-varying one, and a static theory cannot lead to a correct answer. The static theory in such cases can serve, however, as a useful guide to regimes of behavior, and it is so used here.

In the problem with one species only, the entire current is, of course, returned to the injection plane. The nature of the static theory for this case is extremely simple. The potential distribution for electrons is given in Fig. 1, following Fay, Samuel, and Shockley.[5] The same result is obtained for ions with the signs of the potentials and charges reversed. If an electron emitter at potential V_e is at a distance s from the injection plane, considered transparent to electrons

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traveling toward it from the emitter, the current density injected into the region to the right of the injection plane is

$$J_e = -2.3 \times 10^{-6} \frac{V_e^{3/2}}{s^2} = \frac{4}{9} \epsilon_0 \left(\frac{2e}{m_e}\right)^{1/2} \frac{V_e^{3/2}}{s^2} \tag{1}$$

where e is the electronic charge, ϵ_0 is the dielectric constant of free space, and m_e is the mass of the electron, all in mks units. The static theory predicts that the beam will travel a distance x_m beyond the injection plane, at which point the potential is depressed by the electron space charge to the potential of the electron emitter. The beam turns around here and returns to the injection plane, considered opaque to electrons traveling toward the emitter region, where it is collected. Since the total current contributing to the charge in the region beyond the injection plane is twice the injected current, and since the potential distribution is exactly the same as in the emitter region, the distance x_m will be, from Eq. (1), equal to $s/(2)^{1/2}$.

In single-species, finite diode problems very similar to this, in which the static theory of Fay, Samuel, and Shockley predicts a return current, Lomax[6] and Birdsall and Bridges[7] have found that a one-dimensional theory allowing time variations, using exactly the same model that is presented here, predicts an oscillatory solution rather than a static solution. Results very similar to theirs for the problem of Fig. 1 are presented in the following discussion, along with similar calculations for the two-species problem.

The static ion-propulsion problem is illustrated in Fig. 2. As before, an electron emitter at V_e is a distance s from the injection plane. Here an ion beam is injected simultaneously and an ion emitter is pictured at potential V_i a distance x_0 from the injection plane emitting a current density defined by,

$$J_i = 2.3 \times 10^{-6} \frac{V_i^{3/2}}{x_0^2} \left(\frac{m_e}{m_i}\right)^{1/2} \tag{2}$$

where m_i is the ion mass. An important parameter in the two-species case is the velocity ratio

$$\gamma_0 = u_{e0}/u_{i0} \tag{3}$$

where u_{e0} and u_{i0} are the injection velocities of the electrons and ions, respectively, and where $u_{e0} = (2eV_e/m_e)^{1/2}$ and $u_{i0} = (2eV_i/m_i)^{1/2}$. It should be noted that it is not proper to regard these electron and ion diodes as actually existing physically in the space to the left of the injection plane. The only problem discussed here is the one to the right of the injection plane, with prescribed injection velocities and currents defined in terms of the distances and potentials of these equivalent diodes. The static theory[1–4] for this region to the right of the injection plane yields a potential variation of the form indicated in Fig. 2, if the injected currents are equal and if the ions have infinite mass. For equal injection velocities and currents, γ_0 is unity and the potential is constant. This case is that of a perfectly cold drifting plasma with charge neutrality everywhere. If γ_0 is less than unity and there are equal injected currents, there can only be space-average charge neutrality, if the electrons are, on the average, speeded up. This occurs by means of a periodic variation in potential having a period comparable to the equivalent electron diode spacing. Similarly, if γ_0 is greater than unity, the electron velocity must be, on the average, reduced in order to provide charge neutrality, and again this is accomplished by means of a periodic potential variation. At the critical value of γ_0 equal to 2 these periodic potential depressions reach a maximum, with the lowest potential equal to the potential of the electron emitter. The potential can be reduced no further. For γ greater than 2 the static theory can, however, be made to yield charge neutrality by postulating the existence of current loops oscillating in the potential wells shown in Fig. 2, in addition to the main beam of electrons traveling to the right. These loops of current do not contribute to the net current traveling to the right but do contribute to the average charge. By this means, static charge neutrality can, in principle, be accomplished with electrons injected at very much higher velocities than the ions. In practice there is no way to establish these current loops, and this type of solution is not found in the computer results to be presented.

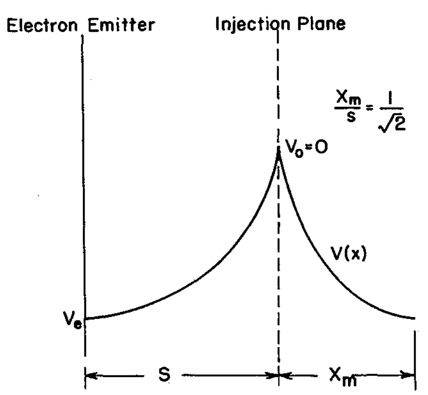

Fig. 1 Potential vs distance variation from static theory for an unneutralized one-dimensional electron beam shot into a field-free space to the right of the injection plane. The electrons all return to the injection plane where they are collected.

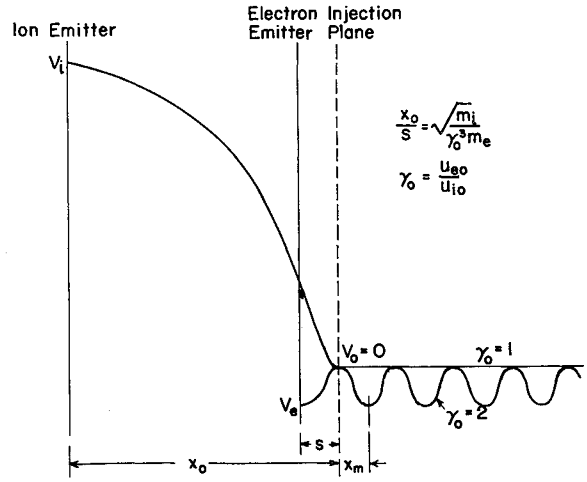

Fig. 2 Potential vs distance variation from static theory for a neutralized beam with equal absolute values of ion and electron current. For γ_0 greater than 2 the potential has the same general shape as shown for $\gamma_0 = 2$, but there are loops of electron current in each potential trough in addition to the current transmitted. No ions are returned.

Although the results to be presented are all for single-velocity injection, some of them are of interest in explaining neutralization of ion beams by electron sources with Maxwellian velocity distributions. The one-dimensional static theory for this case has been discussed in Refs. 1 and 3. The mean thermal electron speed in this case replaces the electron velocity of the single-velocity-injection case. For mean thermal electron speeds less than the ion velocity, the electrons are, on the average, speeded up by means of periodic potential variations to provide charge neutrality, just as in the single-velocity problem. However, the one-dimensional static theory for mean thermal electron speeds greater than the ion velocity is not directly analogous to the single-velocity case, and no periodic solutions are obtained.

The difficulty with a static theory in this case has to do with the fact that many electrons bounce back and forth in current loops that are the full length of the ion beam. This type of trajectory is observed in the computer results obtained here. As has been suggested by Mirels,[3] the system can, in this case, be viewed as a plasma bottle with sheaths that return all of the electrons at both ends. In a finite-diameter system there would be a radial sheath as well, to provide a complete container within which the fast electrons could rebound. Transient measurements by Sellen and Kemp[8] confirm this qualitative picture of the neutralization mechanism when neutralization is accomplished by means of electron emission from a hot wire immersed in the ion beam. An attempt to formulate a static theory, in this case, encounters the difficulty that the finite length of the ion beam at any instant of time determines the size of the container, and this length is changing with time at a rate comparable to the velocity of the electrons contained in the system. Thus, the situation is inherently a transient one. Nevertheless, the qualitative picture proposed by Mirels[3] seems correct at any instant of time, and is confirmed by the transient results presented here.

Another mechanism basic to the process of neutralization is the apparently self-compensating feed system that was found by Sellen and Kemp[8] to be provided by a hot electron emitter immersed in the ion beam. It appears that the one-dimensional dynamic model to be discussed can also help to explain this result by a mechanism that is present with single-velocity injection of electrons at velocities greater than twice the ion velocity. Briefly, it appears that if enough high-velocity electrons are supplied to the system, they will be brought nearly to rest and will form an oscillating sheath; a natural selection process will then take place at this sheath in which enough electrons of the right velocities are selected to provide charge neutrality in the system beyond the sheath. Neutrality is obtained on a space-and-time average basis, with electrons of many velocities and with many types of trajectories being present.

These results are also believed to be of importance in explaining the practical case of neutralization by wires or other electron emitters just outside the beam rather than immersed in it. In this case a considerable initial drift velocity, in addition to their random velocities, can be acquired by injected electrons on their way into the ion beam. Still, as will be shown, neutralization and self-compensating current injection can occur, if enough current is available from the emitter to allow formation of an oscillating sheath from which lower velocity electrons can be selected.

II. One-Dimensional Sheet Model

Following the approach used by Buneman[9] and by Dawson[10] in plasma problems and by Hartree,[11] Lomax,[6] and Birdsall and Bridges[7] in electron beam problems, we take a model in which a number of discrete charge sheets are used to simulate the (nearly) continuous distribution of charges of the physical problem. We are able to compute the self-consistent field of all of the charges and to follow their trajectories in those cases in which they return through a region already traversed, and in other crossover situations. Each charge sheet will enter the region of interest from the injection plane at a specified velocity and will have a charge density ρ_s per unit area. As ρ_s is made smaller and the number of charge sheets used to simulate the continuum of charge is increased, the results of this type of calculation should and do approach a definite answer.

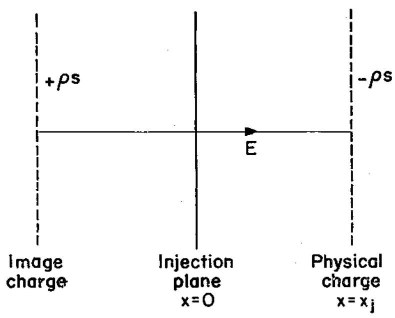

Fig. 3 A single one-dimensional charge sheet near a perfectly conducting plane and its image charge. The force between charges is independent of distance.

Let ρ_{s0} be the charge that would be contained in a volume element of unit area and of length equal to the equivalent diode spacing s carrying the injected electron current density J_e at velocity u_{e0}. Then ρ_{s0}, a charge per unit area, is

$$\rho_{s0} = J_e s/u_{e0} \tag{4}$$

The time may be normalized with respect to T_0, the time to traverse the distance s at velocity u_{e0},

$$T_0 = s/u_{e0} \tag{5}$$

and distance may be normalized with respect to s.

Figure 3 shows a single sheet of charge near a perfectly conducting injection plane and its image charge. The field in the region between the injection plane and the charge at x_j is, for a negative charge at x_j as in Fig. 3,

$$E_x = \rho_s/\epsilon_0 \qquad 0 < x < x_j \tag{6}$$

There is also a force acting on the charge itself at x_j which corresponds to a field

$$E_x = \rho_s/2\epsilon_0 \qquad x = x_j \tag{7}$$

Beyond the charge, to the right,

$$E_x = 0 \qquad x > x_j \tag{8}$$

Now, if instead of a single electron sheet at x_j, there are M ion sheets of charge density ρ_{is} per sheet at x_{il}, and N electron sheets of charge density ρ_{es} per sheet at x_{el}, the force on the jth electron sheet is

$$f_{ej} = -\frac{e\rho_{es}}{\epsilon_0}\left[\frac{1}{2} + \sum_{l=1}^{N} S(x_{el} - x_{ej}) - \frac{\rho_{is}}{\rho_{es}}\sum_{l=1}^{M} S(x_{il} - x_{ej})\right] \tag{9}$$

where

$$S(x - x_j) = 0 \qquad x \leq x_j \tag{10}$$
$$S(x - x_j) = 1 \qquad x > x_j$$

Similarly the force on the kth ion sheet is

$$f_{ik} = -\frac{e\rho_{is}}{\epsilon_0}\left[\frac{1}{2} - \sum_{l=1}^{N} S(x_{el} - x_{ik}) + \frac{\rho_{es}}{\rho_{is}}\sum_{l=1}^{M} S(x_{il} - x_{ik})\right] \tag{11}$$

It is seen that in this one-dimensional model with one boundary at infinity, the force is obtained simply by counting charges to the left and right of the charge in question. The dynamic equations for the jth electron sheet at time t then can be written in terms of an arbitrary time increment Δt, using the assumption of constant acceleration during this time increment as in the work of Birdsall and Bridges.[7] The acceleration is exactly a constant, if there are no charge crossings during this time period:

$$a_{ej} = f_{ej}/m_e \tag{12}$$

$$v_{ej}(t) = v_{ej}(t - \Delta t) + [a_{ej}(t - \Delta t)]\Delta t \tag{13}$$

$$x_{ej}(t) = x_{ej}(t - \Delta t) + [v_{ej}(t - \Delta t)]\Delta t + \tfrac{1}{2}[a_{ej}(t - \Delta t)](\Delta t)^2 \tag{14}$$

The ion equations are:

$$a_{ik} = f_{ik}/m_i \tag{15}$$

$$v_{ik}(t) = v_{ik}(t - \Delta t) + [a_{ik}(t - \Delta t)]\Delta t \tag{16}$$

$$x_{ik}(t) = x_{ik}(t - \Delta t) + [v_{ik}(t - \Delta t)]\Delta t + \tfrac{1}{2}[a_{ik}(t - \Delta t)](\Delta t)^2 \tag{17}$$

It is convenient for the class of problems to be solved here to take $\rho_{es} = \rho_{is}$, simplifying Eqs. (9) and (11). The picture is then one of discrete ion and electron sheets with each sheet having the same magnitude of charge. The sheets may then be regarded as large electrons and ions. It is also convenient to compute the trajectories using a time increment $\Delta t = \Delta T$, where ΔT is the time between the injection times of adjacent electron sheets:

$$\Delta T = T_0(\rho_{es}/\rho_{s0}) = (\rho_{es}/J_e) \tag{18}$$

It would be possible to calculate with finer time increments, of course, but this choice corresponds to a calculation every time a new charge is added to the system, and as will be shown, this is many times per plasma period. Equations (12–17) can be normalized by dividing the acceleration equations by (u_{e0}/T_0), the velocity equations by u_{e0}, and the distance equations by s. If $\rho_{es} = \rho_{is}$, the constant in the normalized electron and ion acceleration equations can then be written in terms of ΔT and T_0 as follows:

$$\frac{e\rho_{es}T_0}{\epsilon_0 m_e u_{e0}} = \frac{2}{9}\frac{\Delta T}{T_0} = \frac{\omega_{pe0}{}^2 s^2}{u_{e0}{}^2}\frac{\Delta T}{T_0} \tag{19}$$

$$\frac{e\rho_{is}T_0}{\epsilon_0 m_i u_{e0}} = \frac{2}{9}\frac{\Delta T}{T_0}\frac{m_e}{m_i} = \frac{\omega_{pi0}{}^2 s^2}{\gamma^2 u_{i0}{}^2}\frac{\Delta T}{T_0} \tag{20}$$

where

$$\omega_{pe0}{}^2 = \frac{e\rho_{s0}}{\epsilon_0 m_e s} = \frac{2}{9T_0{}^2} \tag{21}$$

$$\omega_{pi0}{}^2 = \frac{e\rho_{s0}}{\epsilon_0 m_i s} = \frac{2}{9T_0{}^2}\frac{m_e}{m_i} \tag{22}$$

Here ω_{pe0} is the electron plasma frequency corresponding to the injected volume charge density (ρ_{s0}/s), and ω_{pi0} is the corresponding ion plasma frequency. It is thus evident that trajectory calculations can be made, if γ_0, (m_i/m_e), and $(\Delta T/T_0)$ are specified as parameters. The value of $(\Delta T/T_0)$ sets the fineness of the grain of the approximation to the continuous fluid. As a guide to the value of this parameter, it is interesting to compute the number N_0 of sheets injected during one plasma period based on the plasma frequency at the injected volume density. This may be computed from the time between electron injections ΔT and the plasma period $(2\pi/\omega_{pe0})$ as given in Eq. (21):

$$N_0 = \frac{2\pi}{\omega_{pe0}\Delta T} = \frac{2\pi T_0}{(2/9)^{1/2}\Delta T} = \frac{13.33}{\Delta T/T_0} \tag{23}$$

It will be shown in the following section that an N_0 of about 26 sheets per plasma period gives satisfactory results, corresponding to $\Delta T/T_0 = 0.5$, but that half this number represents too coarse a model in the sense that further subdivision of the sheets leads to noticeably different trajectory patterns.

III. Single-Species Injection

The static solution for this problem is shown in Fig. 1 and predicts that all of the injected current is returned to the injection plane after traveling a distance 0.707 s into the region of interest. The electrons or ions slow to zero velocity at the turnaround point at x_m. This is a Fay, Samuel, and Shockley type A distribution of potential. The only parameter to be varied in the discrete charge-sheet model is $\Delta T/T_0$, if the time increment is set at $\Delta t = \Delta T$. This means that these results are independent of injection velocity and particle mass. Trajectories for values of $\Delta T/T_0$ of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, and 1 have been calculated and the computer allowed to run to a point where a repetitive or nearly repetitive pattern was observed. The results for these values of $\Delta T/T_0$ are given in Figs. 4–8. Each figure has X vs T as coordinates, where

$$X = x/s \tag{24}$$

$$T = t/T_0 \tag{25}$$

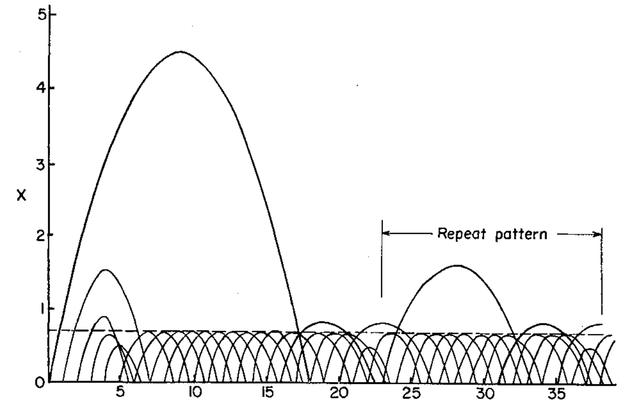

Fig. 4 Trajectories in the distance vs time plane for charge sheets of an unneutralized beam shot into an initially empty space. $(\Delta T/T_0) = 1$. The pattern is the same for either electrons or ions. The steady-state condition is a time-varying one.

It will be seen from Fig. 4 that there is an initial transient corresponding to the time taken for the first sheet to go into the initially empty space and return. The first sheet penetrates farther and returns later than the sheets injected after a number of sheets are already in the space. Our main interest centers on the time period after this initial transient has died out, and Fig. 5 shows an enlarged view of the part of Fig. 4 after the first sheet has returned. Figures 6–8 show corresponding time periods for smaller values of $(\Delta T/T_0)$. The pattern of trajectories for $((\Delta T/T_0) = \frac{1}{8}$ and $\frac{1}{4}$ is substantially identical. Evidently $(\Delta T/T_0) = \frac{1}{2}$ is about the dividing line for an adequate degree of fineness. For smaller $(\Delta T/T_0)$, little change occurs. In more complex problems there is an important advantage in computer time if $(\Delta T/T_0)$ is made as large as possible and $(\Delta T/T_0)$ of $\frac{1}{2}$ has been used in some more complex problems to be discussed.

The most obvious feature of all of these trajectories is that they are oscillatory and do not approach a steady state in which all trajectories are identical. They do, however, oscillate about the static solution as an average. The number of particles returning is time modulated, so an a.c. current to the injection plane exits.

Another feature of interest is the frequency of oscillation. When compared with the electron plasma frequency corresponding to the density ρ_{s0} which is the plasma frequency at the injection plane, from Eq. (21), we find the observed angular frequency to be about twice this value. If the charge density is calculated from the trajectory plot, it is found

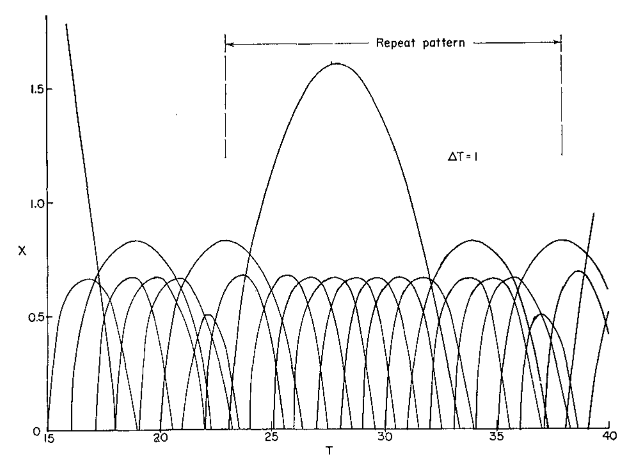

Fig. 5 Trajectories after the initial transient for the same conditions as Fig. 4 but drawn to a different scale.

to be greatest at the times at which the electrons penetrate the smallest distance into the space and at these points it corresponds to a plasma frequency very nearly equal to the measured oscillation frequency. These comparisons are in agreement with some related calculations in other cases for electron diodes with finite spacing by Birdsall and Bridges.[7]

This basic effect of an oscillating potential minimum represents, in a sense, nature's way of avoiding the infinite charge density predicted in the static theory at the turnaround point. The correct solution to this problem is thus a time varying solution rather than a static one. The effect is difficult to observe experimentally with electrons because the electron plasma frequency is high, and it is difficult to observe directly in the current leads to the electrodes. It is a straightforward measurement with ions, however, and experimental results confirming this computer result have been obtained by Sellen and Shelton[12] in an unneutralized cesium ion beam experiment with a gridded gun. They observed an oscillation frequency that varied with current in the right way and was of the right order of magnitude. Their physical experiment was slightly different from our computer experiment in that the particles were not all collected on the injection plane on the first pass in the physical experiment; some were returned into the cathode-grid region. Other modes of oscillation were thus possible, but at least it is clear that a static solution is not obtained experimentally in either case and that the experimental results are qualitatively in agreement with the theory presented here.

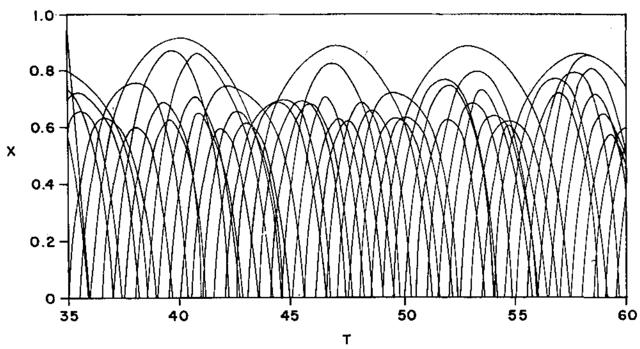

Fig. 6 Trajectories for $(\Delta T/T_0) = 0.5$ drawn to the same scale as Fig. 5.

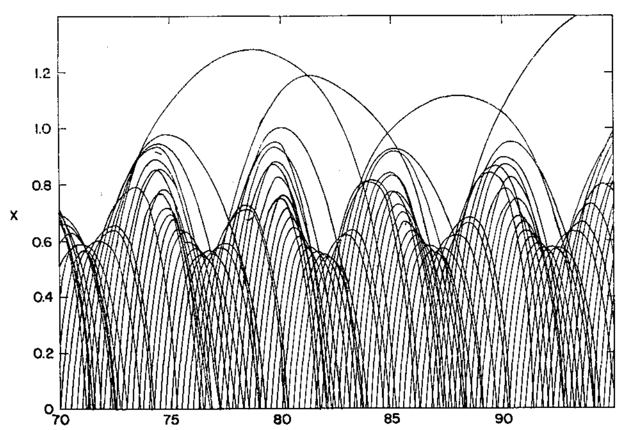

Fig. 7 Trajectories for $(\Delta T/T_0) = 0.25$ drawn to the same scale as Fig. 5.

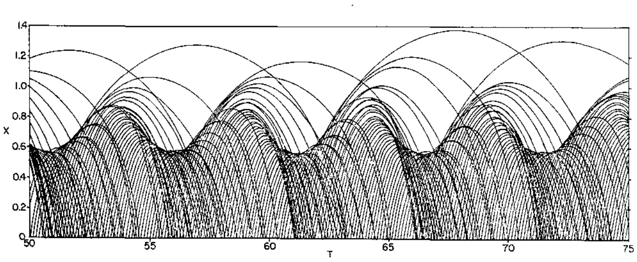

Fig. 8 Trajectories for $(\Delta T/T_0) = 0.125$.

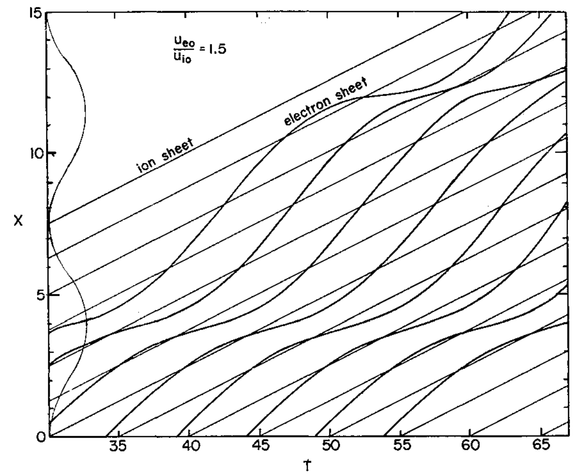

Fig. 9 Trajectories of electrons and ions in a neutralized beam with equal absolute values of injected electron and ion currents for $\gamma_0 = 1.5$ and $(m_i/m_e) = 1800$. Static theory predicts the potential variation at the left and the computed trajectories follow this pattern.

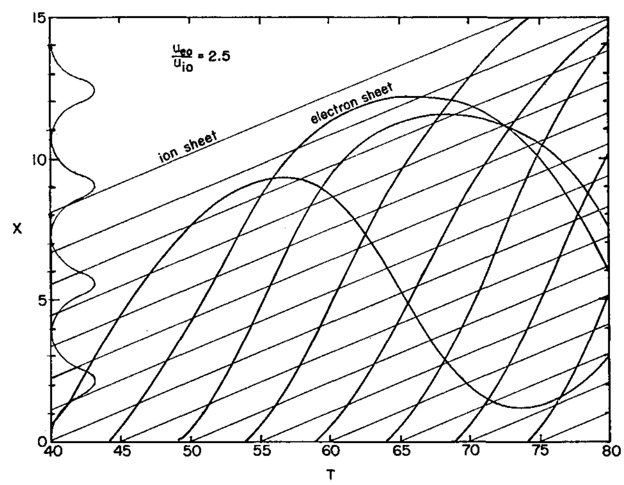

Fig. 10 Trajectories of electrons and ions in beams with injected current neutrality for $\gamma_0 = 2.5$ and $(m_i/m_e) = 1800$. The computed trajectories do not follow the pattern of the static-theory potential variation at the left.

IV. Electrons and Ions with Equal Absolute Values of Injected Current

The static solution for this problem is shown in Fig. 2 and is a periodic potential variation with distance, with all particles injected being transmitted if γ_0 is less than 2. For γ_0, the ratio of electron injection velocity to ion injection velocity, equal to or greater than 2, a return current of electrons is predicted; but the net electron current, injected minus return, is equal to ion current. We therefore expect, by analogy with the solutions with a single species, to find that the discrete sheet model will predict oscillations when the static theory predicts a return current, and this is what is found. In the static theory of Ref. 1, charge neutralization is obtained for γ_0 greater than 2 by allowing electron current loops in the potential troughs of Fig. 2. The first trough, adjacent to the injection plane, is the one that provides the return current of electrons, just as in the single-species problem. Somehow it is imagined that an infinite array of loops

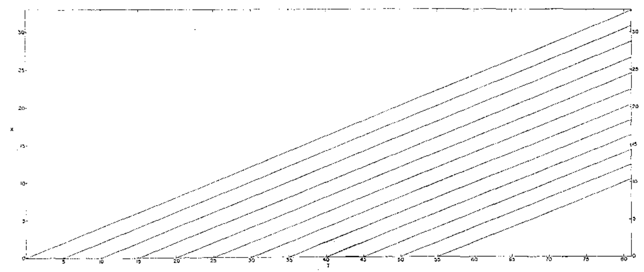

Fig. 11 Ion trajectories for the case of Fig. 10 drawn to a different scale. $\gamma_0 = 2.5$; $(m_i/m_e) = 1800$. Distance X vertically and time T horizontally. Only every fifth trajectory out to $T = 55$ is shown.

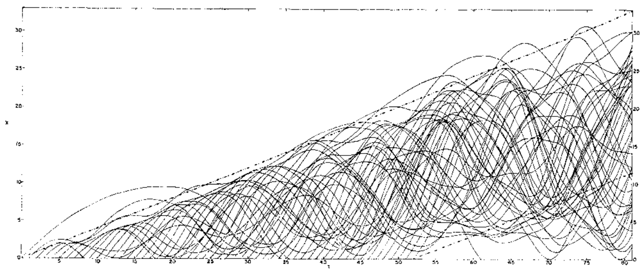

Fig. 12 Electron trajectories that go with the ion trajectories of Fig. 11. $\gamma_0 = 2.5$; $(m_i/m_e) = 1800$. Some electrons return to the injection plane. Distance X vertically and time T horizontally. Ion trajectories for ions injected at $T = 0$ and $T = 52$ are shown as the nearly straight lines with circles.

has been established in the troughs beyond the first one. This picture is obviously unrealistic and is not the way one would expect a system to establish itself, given equal injected currents and an initially empty region to the right of the injection planes, nor is this the result that is obtained. As will be shown, however, the system does behave as predicted by the static theory for γ_0 less than 2.

The trajectories from the sheet model can be found, if $(\Delta T/T_0)$, (m_i/m_e), and γ_0 are specified, for the case of J_i equal to minus J_e. Figures 9–12 are trajectories for $(\Delta T/T_0)$ equal to $\frac{1}{2}$ and (m_i/m_e) equal to 1800. In Fig. 9 γ_0 is equal to 1.5, and in Figs. 10–12, γ_0 is equal to 2.5. The static theory prediction for potential as a function of distance is shown at the left of Figs. 10 and 11. The static theory of Ref. 1 is not substantially altered by the inclusion of a large but finite ion mass ratio, such as 1800, if current loops are postulated as a means of obtaining charge neutrality.

If γ_0 is equal to unity, all particles are transmitted without any variation of potential with distance. The trajectories for the sheet model agreed with this result of static theory, but are not shown. Figure 9 is a trajectory plot for $\gamma_0 = 1.5$ in which the electron sheets near the beam front are not shown. Here a periodic velocity variation occurs in space but not in time, just as in the static theory.

For $\gamma_0 = 2.5$, velocity variations occur both in space and time and the results are radically different from the predictions of static theory. The same fraction of the total number of computed trajectories is plotted in Fig. 10 as in Fig. 9, and it is evident that the electrons are not paying any attention to the potential variation plotted at the left that was computed from static theory, including current loops. Figure 11 gives a more complete set of ion trajectories for the same case of $\gamma_0 = 2.5$. The ion velocity is only imperceptibly altered over the time period for which calculations were made. Figure 12 shows the electron trajectories corresponding to the ion trajectories of Fig. 11. Electrons and ions were continually injected after $T = 52$, but the plotting was stopped at this point to permit distinguishing the electrons injected during this time from those injected later. Thus we have, qualitatively, the plasma bottle picture for most of the fast electrons, i.e., they bounce back and forth between the two ends of the ion beam. There are also many slower electrons present that oscillate in both time and space with smaller amplitude excursions. As the beam grows in length the amplitude of the maximum electron excursions extends farther and farther, also in accordance with the plasma bottle picture.

It will also be seen that there are some electrons being returned to the injection plane, but the percentage of those that are turned around that actually go all the way back to the injection plane without being turned around a second time is small. We have started with injected current neutrality, but after there have been electrons returned in this situation, one would ultimately expect some ions to return in order to maintain net charge neutrality. To pursue this question further, a lower ion-to-electron mass ratio was used to allow a greater range of phenomena to be observed without excessive computer time. It is evident that this problem requires an observation of a linearly increasing number of particles as a function of time, so that the longer that time runs, the more it costs per unit time to continue the calculation. Also it is evident that the scale of the ion phenomena, for large ion-to-electron mass ratios, will be larger for larger mass ratios. On this basis a mass ratio of 50 was chosen for calculations in which ion trajectories were of interest. It is believed that this number is large enough in comparison with unity to insure that qualitatively similar results will be obtained for larger mass ratios, and this was borne out by one calculation of ion trajectories shown in Fig. 13 which was for a mass ratio of 1800 and a velocity ratio of 5. A change in mass ratio to 50 did not significantly alter the form of the trajectories shown. A similar computation with a mass ratio of 50 and a velocity ratio of 2.5 also gave the same pattern, but on a larger scale. It is seen that there is a time period at the beginning of Fig. 13 corresponding to the situation in Fig. 11. For longer times the ions do, however, turn back in substantial numbers, but with some fraction continuing on. It is also interesting to note that the ions near the foremost end of the beam have been accelerated and are now going faster than their initial velocity. This feature of the trajectories can be explained in terms of the effect of the electrons in the electron sheath that extends out ahead of the foremost ion. These electrons will exert a force on the ions following them that will accelerate these foremost ions to a greater velocity. Some idea of the form of this electron sheath can be obtained from the trajectories of Fig. 12.

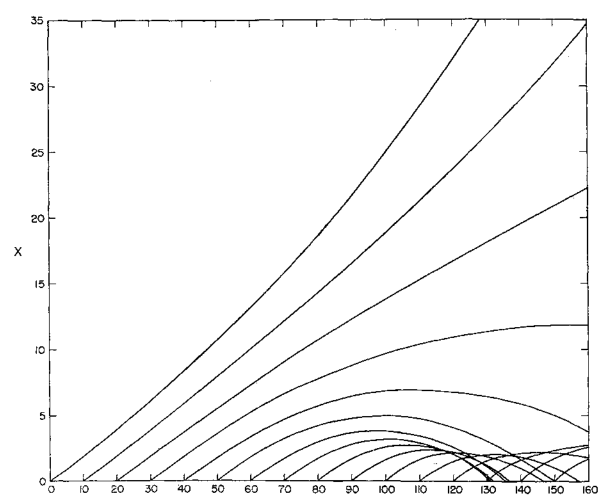

Fig. 13 Ion trajectories with injected current neutrality for $\gamma_0 = 5$ and $(m_i/m_e) = 1800$. Both electrons and ions return to the injection plane. Distance X vertically and time T horizontally.

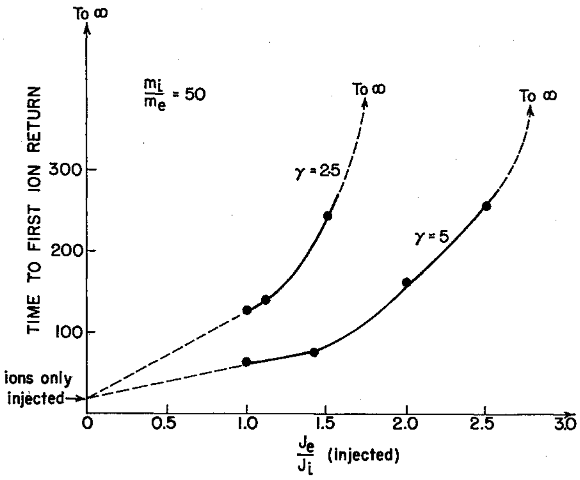

Fig. 14 Time to first ion return as a function of the ratio of injected electron current to ion current, for $\gamma_0 = 2.5$ and $(m_i/m_e) = 50$. When this current ratio is increased, the ions stay in the system longer. If the current ratio is increased far enough, the ions do not return at all.

It is also interesting to note that once an ion is slowed to zero velocity and turned around, as in Fig. 13, it goes all the way back to the injection plane, unlike the situation for electrons in Fig. 12.

This over-all picture, obtained with equal absolute values of injected electron and ion currents, agrees with a static theory picture of neutralization for γ_0 less than 2 and shows electron behavior in accordance with the plasma bottle picture of neutralization for γ_0 greater than 2. However, complete neutralization is not actually obtained for γ_0 greater than 2 and equal injected electron and ion currents, as is evidenced by the return of ions to the injection plane if the computation is continued long enough.

V. Electrons and Ions with Unequal Values of Injected Current

The preceding section has described the trajectories obtained with equal values of injected current, starting with an initially empty space to the right of the injection plane. The failure to obtain neutralization was first indicated by the return of a few electrons to the injection plane. One fairly obvious approach in attempting to keep the system neutral is to replace each electron that is lost by injecting a new one (in addition to the uniform current being introduced) whenever an electron is returned. This would amount to making the injection plane a mirror for returning electrons. Rather than pursue this possibility, we have simply increased the injected current by increasing the number of sheets injected per unit time, keeping the injection rate uniform in time.

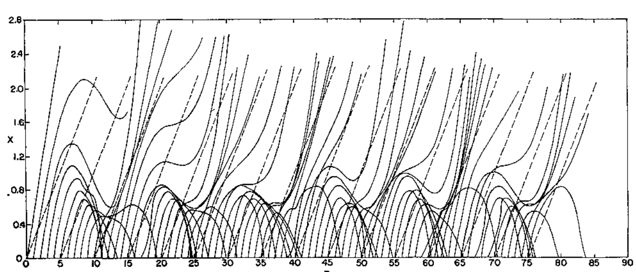

Fig. 15 Trajectories of electrons and ions near the injection plane for $\gamma_0 = 2.5$, $(m_i/m_e) = 50$, and $(-J_e/J_i) = 2$, a value above the critical current. An oscillating electron sheath supplies electrons at low velocities. Distance X vertically and time T horizontally.

The first calculation was made for the two cases γ_0 of 2.5 and 5, and for current ratios in the amounts required by static theory to provide current neutrality. These amounts are found from the static theory of Ref. 1 to be

$$-\frac{J_e}{J_i} = 1 + \frac{\gamma_0 - 2}{\gamma_0 + 2}$$

and so correspond to electron-to-ion current ratios of 1.1111 for γ_0 of 2.5, and 1.4286 for $\gamma_0 = 5$. The computer results for returned electron current with equal injected currents were not far from these values; thus the idea that neutralization might be obtained with these electron-to-ion current ratios is at least plausible. In fact, the result was simply that the time that elapsed before the first ion returned was increased, and neutralization was not obtained. The next step that suggests itself is to try injecting still more electron current. The results are indicated in Fig. 14, which is a plot of the time to the first ion return as a function of the injected electron-to-ion current ratio. An infinite time to the first ion return would represent a neutralized beam. As can be seen, a very significant change in the slope of these curves is obtained as the current ratio is increased, suggesting that, indeed, an infinite time can be obtained with quite modest current ratios above a critical value that depends on γ_0. A value of the current ratio of 2 was tried for γ_0 of 2.5, which is a case for which one would expect neutralization from the shape of these curves. Two important results were obtained:

1) The ion trajectories were of a qualitatively different nature from those obtained for lower current ratios, such as those in Fig. 13. In the neutralized case, ions were slowed to a minimum nonzero velocity and then were speeded up again, then slowed down, etc., in a fairly repetitive pattern, instead of being either continuously speeded up or slowed down and returned. A similar result was obtained for a current ratio of 3, and no ions were returned in either case out to $T = 450$.

2) The electron trajectories near the injection plane were of a character reminiscent of the single-species results, as shown in Fig. 15, which is a trajectory plot for γ_0 of 2.5, a mass ratio (m_i/m_e) of 50, and a ratio of injected electron-to-ion current of 2.

The interesting feature of Fig. 15 is the oscillating potential minimum that occurs in the same general form as in the single-species problem. Here, however, electrons are drawn into the ion beam from this oscillating sheath at velocities and in quantities as needed to provide neutralization. Although the electrons all start out with the same high velocity, those that emerge from the oscillating sheath typically have lower velocities nearer the ion velocity. It is also easy to see the mechanism of natural electron selection at work; whenever a new electron is needed to go with an additional ion that has been injected, there are plenty of slow electrons available that are just about ready to return to the injection plane. As needed, electrons are removed from this virtual electron emitter and drawn into the ion beam. Thus the net electron and ion currents beyond the oscillating sheath have equal absolute values, as needed for complete neutrality.

VI. Conclusions

A type of one-dimensional theory has been presented that is valid for transient, time-varying, drastically nonlinear conditions in electron and ion beams injected into an initially field-free space. The results of a series of computer experiments using this theory have been obtained. Only initially single-velocity beams with ion-to-electron mass ratios of 50 or greater have been treated. The main variables have been found to be the ratio of electron-to-ion velocity and the ratio of electron-to-ion current at the injection plane.

There are two main regimes that characterize the behavior of the system as the ratio of electron-to-ion current is varied. In the first regime there are not enough electrons injected to neutralize the ion space charge and some ions return to the injection plane. This regime may be characterized as the "electron-starved" regime. Only partial neutralization is obtained; the degree of neutralization can be characterized by the fraction of the ion current that is returned to the injection plane, and by the time taken for the first ion to return.

In the second regime there are more than enough electrons to neutralize the ion space charge and all ions are transmitted to infinity; there is complete neutralization. In this second regime there are excess electrons injected and these excess electrons return to the injection plane. They are reflected from an oscillating electron sheath that feeds the right number of electrons at the right velocity into the ion beam to provide complete neutralization. This second regime may be characterized as the "oscillating-electron-sheath" regime.

Figure 16 shows these two regimes in the (J_e/J_i) vs γ_0 plane. The exact boundary has not been determined with precision, the heavy dots corresponding to the only points that are known. A solid-line boundary between the two regimes is shown from γ_0 equal to 0 out to γ_0 equal to 2. Along this line neutralization is complete but is obtained by a different mechanism, the mechanism of the static theory in which electrons are all speeded up or slowed down at the same points in space to provide charge neutrality. Along this line we may characterize the system as being in the "static neutralization" regime. It exists only along this line. At all other points in the plane an essentially time-varying type of process occurs, with electrons being returned to the injection plane.

It is believed that these results, although obtained from a one-dimensional model with single velocity injection, give considerable insight into the mechanisms of neutralization in practical ion-propulsion engines. In a neutralization system in which an electron emitter is immersed in the ion beam, it is to be expected that the effect of random velocity injection will be a somewhat blurrier oscillating sheath than is provided by the single-velocity system studied here. The typical practical neutralization system in which the electron emitter neutralizer element is somehow shielded from the direct path of the ion beam may be expected to operate in almost exactly the manner described here by a single-velocity injection theory. This statement is based on the fact that in such practical neutralization systems the electrons can enter the ion beam at even higher velocities than those corresponding to the emitter temperature, because they are accelerated into the ion beam by a local potential gradient. The present theory suggests that they would form an oscillating sheath and again be slowed to near zero velocity before entering the ion beam. The present theory also allows a prediction of the amount of available excess current that must be provided in order to give complete neutralization in the "oscillating-electron-sheath" regime, if the electron injection velocity can be estimated.

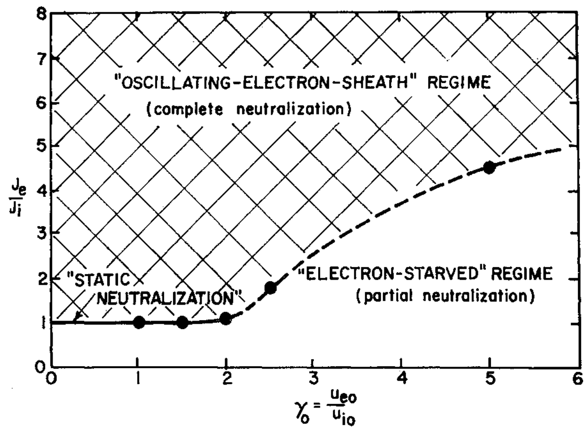

Fig. 16 Ratio of injected electron current to ion current for neutralization as a function of γ_0. Three regimes of operation are shown. The "static neutralization" regime is confined to the solid line up to $\gamma_0 = 2$. In the "electron-starved" regime ions return as in Fig. 13. In the "oscillating-electron-sheath" regime neutralization is obtained by the mechanism of Fig. 15.

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